

# de Sitter equilibrium as a fundamental framework for cosmology<sup>1</sup>

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**Abstract.** Cosmology might turn out to be the study of fluctuations around a “de Sitter equilibrium” state. In this article I review the basic ideas and the attractive features of this framework, and respond to a number common questions raised about the de Sitter equilibrium picture. I show that this framework does not suffer from the “Boltzmann Brain” problem, and relate this cosmological picture to recent work on the “clock ambiguity”.

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## 1. INTRODUCTION

Most approaches to theoretical cosmology ultimately require a framework which can quantify the relative probabilities of different cosmological scenarios. For example, the widely held belief that cosmic inflation gives an account of the initial state of the big bang which is better or “more natural” than models without inflation can only be substantiated within such a framework. One could hope that such a framework would allow the various features of inflation that seem intuitively appealing to directly enhance the probability assigned to cosmologies with inflation in a quantifiable way.

A great many models of cosmic inflation yield a picture called “eternal inflation” [1] where inflation continues forever in (exponentially) increasingly large regions, periodically reheating in isolated regions known as “pocket universes” which can look something like the big bang cosmology we observe (for some reviews and a sample of recent work see [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]). It has long been hoped that the eternal nature of eternal inflation would allow the predictions to be independent of any initial conditions at the start of inflation, and thus yield stable results that could be regarded as the robust predictions of the inflationary cosmology.

So far things have not turned out this way. Eternal inflation typically leads to a variety of types of pocket universes, each produced in infinite quantities. To predict which type of pocket universes are most likely one must regulate these infinities in some way. To date, there is no physically determined (or even universally agreed upon) way to regulate the infinities, and discussions of this matter usually invert the question to: “Which of

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the various possible regularization schemes yield predictions that actually match what we observe?” (Many do not.) Posing the question this way may be realistic and even interesting in the absence of something better. But as long as one is framing the problem in these terms one is, at least for the time being, abandoning the prospect of actually making concrete predictions from inflation. If one embraces the string theory landscape picture the problem is only made greater by an unspecified multitude of possible inflatons experiencing eternal inflation. One might hope that a better understanding of the underlying physics would eventually regulate the infinities for us and lead to robust predictions (perhaps holographic considerations[13, 14, 15, 12] and/or other approaches to carefully developing the formalism[16, 17, 18, 19, 20] will ultimately help with this). Another possibility is that the current predicament is a reflection of serious problems with the whole framework of eternal inflation which will prevent it from ever gaining predictive power.

This paper is about an alternative framework for assigning probabilities in cosmology which I currently find more promising than eternal inflation. This framework, which I call the “de Sitter Equilibrium” (dSE) cosmology has been examined in two previous papers[21, 22]. In this article I briefly review the dSE picture and call out the key reasons I find this framework attractive. I then comment on some of the questions that have come up regarding dSE since the earlier work appeared. I also offer a quantitative estimate that illustrates how the dSE picture can evade the so-called “Boltzmann Brain” problem. I then discuss how the dSE framework might relate to my recent work on the “clock ambiguity” which at least superficially might appear to be taking things in an orthogonal direction.

## 2. THE DSE FRAMEWORK

### 2.1. General Features

Intuitively, one can think of equilibrium for a system as the state a system achieves if left without external intervention for an arbitrarily long time. By this standard, de Sitter space can be thought of as the equilibrium state achieved by any system which obeys Einstein gravity with a positive cosmological constant  $\Lambda$ . This point requires the cosmological constant to be truly constant, not the energy of a false vacuum that would ultimately decay through tunneling processes[23] or other instabilities[24, 25].

If our universe does have a true (perfectly stable) positive cosmological constant the correct long-term description of the universe would be a state closely approximating de Sitter space. The Hawking temperature of de Sitter space would create fluctuations which could (very rarely) be sufficiently large to cause a temporary deviation into a state that does not look at all like a classical de Sitter space.

Under these conditions the field of theoretical cosmology become the study of the full range of fluctuations out of a de Sitter background, and the interpretation of these different fluctuations as possible cosmological scenarios. One very attractive feature of this picture is that probabilities are assigned to fluctuations in an equilibrium state without any reference to “initial conditions”, a concept which is meaningless in a system

eternally in equilibrium. For fluctuations in an equilibrium state, it is just the laws of physics (the Hamiltonian) that determine the probabilities of various fluctuations. Another attractive feature of the dSE picture is that the presence of a horizon surrounding any observer in de Sitter space leads to quantitative treatments that appears much more naturally finite, in contrast to the infinities which seem to fundamentally plague eternal inflation.

Of course, the main prediction of the dSE picture might be taken to be that the universe should be observed in a pure de Sitter equilibrium state. The viability of dSE as a framework for theoretical cosmology depends crucially on one's willingness to impose the thermodynamic arrow of time as a condition, rather than something that must be predicted as a universal and eternal feature of one's theory. Many authors (although not all [26, 27]) have indeed found it reasonable to use the arrow of time as a condition [28, 29, 30, 31]. This could come about through some sort of anthropic argument related to how critical the arrow of time is for the functioning of observers like us, or it could be a much more narrowly formulated choice to use the fact that we observe an arrow of time as a condition to pose conditional probability questions about the universe.

## 2.2. de Sitter Entropy

An observer in a de Sitter space with positive cosmological constant  $\Lambda$  is surrounded by an event horizon of radius  $R_\Lambda$  given by

$$R_\Lambda^{-2} = \frac{\rho_\Lambda}{3m_p^2} = \frac{\Lambda}{3}. \quad (1)$$

Throughout we use  $\hbar = c = k_B = 1$  and  $m_p^2 \equiv l_p^2 \equiv 1/8\pi G$ . As with the black hole case, the horizon is associated with an entropy

$$S_\Lambda = \pi \frac{R_\Lambda^2}{l_p^2} \quad (2)$$

Gibbons and Hawking (who were the first to propose and study de Sitter entropy [32]) showed that when other objects (in particular black holes) are put in a de Sitter space the overall entropy goes down. Specifically, when a black hole with entropy  $S_{BH}$  is placed inside a de Sitter space with entropy  $S_\Lambda$  the horizon of the de Sitter space shrinks so that the entropy of the combined system is

$$S_\Lambda \rightarrow S_\Lambda - \sqrt{S_\Lambda S_{BH}} + S_{BH} \approx S_\Lambda - \sqrt{S_\Lambda S_{BH}}. \quad (3)$$

Even though the black hole entropy has added to the total, the decrease due to horizon shrinkage (2nd term) is greater than the increase due to the additional entropy of the black hole (for black holes small enough to fit in the de Sitter horizon in the first place), leading to a net entropy decrease. Any localized matter will look like a black hole sufficiently far away and will decrease the horizon entropy accordingly while adding less

entropy than would be the case for the true black hole. Similar arguments can be made about adding a more uniform radiation field to de Sitter space, and thus the statement that the de Sitter entropy is the maximum possible entropy for a system with a stable cosmological constant appears to be quite robust. This maximal entropy feature is one of the reasons de Sitter space appears fit to be regarded as an equilibrium state.

### 2.3. Recurrences

Both previous papers on dSE cosmology have made use of the following picture: The de Sitter space is regarded as a finite system in a Hilbert space of dimension

$$N_\Lambda \equiv e^{S_\Lambda} . \quad (4)$$

One can then make the ergodic argument that a particular fluctuation which is consistent with  $N_F$  microstates occurs with probability

$$P_F = \frac{N_F}{N_\Lambda} \equiv \frac{t_F}{t_R} . \quad (5)$$

The finite system is expected to experience recurrences on a time  $t_R \propto N_\Lambda$  and the system spends a time  $t_F \propto N_F$  in the fluctuation, which is the origin of the last equality in Eqn. 5. Furthermore, one can infer  $N_F$  from the minimum entropy  $S_F$  exhibited by the entire system during the fluctuation, and thus write

$$P_F = \frac{N_F}{N_\Lambda} \equiv \frac{e^{S_F}}{e^{S_\Lambda}} . \quad (6)$$

If the fluctuation is a small localized mass concentration in the de Sitter background, then (using Eqn. 3)

$$P_F = \exp \left\{ -\sqrt{S_\Lambda S_{BH}} \right\} \quad (7)$$

where  $S_{BH}$  is the entropy of the black hole with equivalent ADM mass to the localized fluctuation.

### 2.4. Probabilities for fluctuation into our observed Universe

The formalism being outlined here should in principle serve as a means to calculate the probability of any fluctuation out of the de Sitter equilibrium. Of particular interest are the probabilities for a fluctuation into an inflating state, a fluctuation into a standard big bang cosmology without inflation, and a fluctuation into a ‘‘Boltzmann Brain’’[22]. If the probability for a fluctuation to enter cosmic inflation is much higher than the others, the formalism supports the prevailing beliefs in theoretical cosmology. Otherwise, the formalism is in conflict with these beliefs. In what follows, we will simply assume that the physical degrees of freedom include an inflaton and other fields suitably chosen and coupled to allow the standard cosmological picture: *inflation*  $\rightarrow$  *reheating*  $\rightarrow$  *standard big bang* (SBB) to be a possible behavior of the system.

### 2.4.1. Probabilities for the Farhi Guth Guven process

In [22] Sorbo and I (AS) studied the formation of an inflating universe from dSE via the following process: A small seed fluctuation of localized matter forms with probability  $P_c$  which then has probability  $P_q$  of fluctuating further into an inflating state. We used Eqns. 3 and 7 to calculate

$$P_c = \exp\left\{-\sqrt{S_\Lambda S_S}\right\} \quad (8)$$

where  $S_S$  is the entropy of the black hole with the same ADM mass as the fluctuation. The (necessarily quantum) probability that the local fluctuation excites the inflaton field and tunnels into an inflating state is given by

$$P_q = \exp\left\{-\pi\left(\frac{R_I}{l_P}\right)^2\right\} \approx \exp\{-S_I\} \quad (9)$$

where  $R_I$  is the de Sitter radius during inflation

$$R_I^{-2} \equiv \frac{\rho_I}{3m_P^2} \quad (10)$$

and  $\rho_I$  is the energy density during inflation. This two-stage process is known as the Farhi Guth Guven process [33, 34]. Combining Eqns. 8 and 9 gives a probability

$$P_I \equiv P_c P_q = \exp\left\{-\sqrt{S_\Lambda S_S} - S_I\right\} \quad (11)$$

for entering into an inflating state.

One should compare this with the probability of fluctuating into the standard big bang without inflation. One can think of the formation of the SBB without inflation as the time reverse of the process of “heat death” of the SBB into de Sitter space. Seen that way the probability of fluctuating straight into the SBB is given by equating the entropy of the fluctuation ( $S_F$  in Eqn. 6) to the entropy of the observed universe  $S_{SBB} \approx 10^{100}$  (or perhaps one should use  $S_{SBB} \approx 10^{85}$ , the entropy the observed universe had in the radiation era). Either way, the probability for fluctuating into the SBB without inflation (from Eqn. 6) is exponentially lower than getting there via inflation (Eqn. 11)<sup>2</sup>

$$\frac{P_I}{P_{SBB}} = \exp\left\{S_\Lambda - \sqrt{S_\Lambda S_{SBB}}\right\} \gg 1 \quad (12)$$

As discussed at length in [22], this calculation appears to validate the standard picture of modern cosmology, where inflation is highly favored over other scenarios.

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<sup>2</sup> This calculation of  $P_{SBB}$  follows the calculation of this quantity in [21]. In [22] we considered a different expression which has a less straightforward motivation (and one which I find less compelling at the time I write this) One could use either expression for  $P_{SBB}$  in what follows without changing any of the key points.

### 2.4.2. DKS probabilities

In [21] Dyson Kleban and Susskind (DKS) argued that a universe undergoing inflation looks a lot like a de Sitter space, but with  $\rho_\Lambda = \rho_I$ , the energy density of the inflaton field during inflation. In parallel with Eqn. 2 one can define

$$S_I = \pi \frac{R_I^2}{l_P^2} \equiv \frac{\pi}{l_P^2} \frac{3m_p^2}{\rho_I} \quad (13)$$

and based on holographic considerations DKS argued that Eqn. 13 should give the entropy of the *entire universe* during inflation. When seen in that light, the probability for a fluctuation leading to inflation is given by

$$P_I = \exp \{-(S_\Lambda - S_I)\} \quad (14)$$

giving

$$\frac{P_I}{P_{SBB}} = \exp \{-(S_{SBB} - S_I)\} \ll 1 \quad (15)$$

Since inflation is disfavored in this picture it runs totally against the modern picture of cosmology. Furthermore DKS argued that, even ignoring inflation there are lots of cosmologies that would be exponentially favored over the observed one (for example, cosmologies with a slightly higher cosmic microwave background temperature and thus a larger value  $S'_{SBB} > S_{SBB}$ ).

## 3. ISSUES AND DISCUSSION

In the this and subsequent sections I will comment on some issues that have been raised in various informal conversations that have taken place since the publication of [22]. Many are related to the differences between the AS and DKS calculations outlined in section 2.4.1 and 2.4.2, so that is a good place to start.

### 3.1. dSE as a heat bath

In the AS approach the de Sitter space is seen as a large system for which the fluctuation involves only a small fraction of the entropy. When the seed forms and fluctuates into an inflating universe, the full physical system describes *both* the de Sitter space (with the fluctuation present, which just looks like a small black hole to an observer elsewhere in the de Sitter space) *and* separately the inflating state, destined to reheat and form the SBB.

This picture depends critically on the belief (not universally shared[35]) that standard features of quantum mechanics should extend to quantum gravity so as to allow a wavefunction that describes one semiclassical spacetime to fluctuate into one that describes more than one semiclassical spacetime represented by different physical degrees of freedom. Thus one would expect the wavefunction following the FG process to be given by

a combination of two wavefunctions, one describing the perturbed background de Sitter space (expressed in one subspace) and one describing the inflating state (expressed in another subspace). As the inflating state reheats, goes through the SBB phase and eventually achieves “heat death” in the final approach to the background de Sitter space, the two parts of the wavefunctions would represent the same physical state and the system would again describe only one semiclassical spacetime, the equilibrium de Sitter space.

To track the entropy of this process, one must evaluate the entropy of the entire system (including both semiclassical spacetimes). From that perspective, the fact that the actual fluctuation that starts inflating has a small entropy makes such a fluctuation more likely, because it removes less entropy from the background de Sitter space than a larger fluctuation would. In other words, the entropy of the entire system makes a smaller drop to start inflation than it would to fluctuate directly into the SBB cosmology, and that makes inflation more likely.

By contrast, the DKS calculation assigns an entropy  $S_I$  to the entire universe once inflation starts. In that approach, the smaller  $S_I$ , the greater the entropic cost of fluctuating into an inflating state, and generally the cost is much greater to fluctuate into inflation rather than directly into the SBB.

At this point whether one chooses the FGG analysis or DKS depends on what one believes about the way a fundamental theory of quantum gravity should work (whether, for example, the holographic interpretation of inflation used by DKS is appropriate). In this paper I focus on the FGG process, which results in a cosmological picture which I find quite attractive.

Interestingly, the holographic analysis that leads to Eqn. 14 in the DKS approach can be duplicated by the Coleman De Luccia[23] (CDL) tunneling process. The CDL process leads to the same quantitative tunneling probability for entering an inflating state from the background or “fundamental” de Sitter space. However, if both the CDL and FGG processes are allowed, FGG will win because it is much faster[36]. Some have expressed skepticism that the FGG process is physically allowed. If FGG could be eliminated, then the tunneling analysis will be dominated by CDL and thus give the same result as the DKS analysis.

### 3.1.1. *Issues with the FGG process*

Here are a few issues some have with the FGG process, along with my thoughts in response:

*Ill-defined path integral:* The path integral methods used by FGG have some curious properties, including a Euclidean interpolating solution that is not a manifold. This has led to some skepticism that perhaps the FGG process is not valid. However, Fischler Morgan and Polchinski (FMP) [37, 38] used a Hamiltonian approach to calculate the same process which does not have any similar technical peculiarities, and the results were the same (this is why the FMP approach was used in [22]).

*The  $m \rightarrow 0$  limit:* In the limit where the mass of the seed fluctuation that initiates the FGG process is taken to zero, the process still proceeds at a finite rate. Some cite this as a problematic feature that suggests the FGG process is unphysical. Personally, I would

expect the  $m \rightarrow 0$  behavior is an artifact of the thin wall approximation (used by both FGG and FMP) which is likely to break down at some finite value of the seed mass (I consider a lower bound on the seed mass based on such considerations in section 4 below).

*AdS/CFT calculations:* Freivogel *et al.* [39] analyzed something like the FGG process using AdS/CFT techniques. They showed that the FGG process they considered violates unitarity and thus they argued the FGG process is not physically allowed. However, AdS/CFT analysis is not able to describe processes that allow the boundary of the bulk to fluctuate. While some believe this is a reflection of the fundamental nature of quantum gravity, another interpretation is that this means that an AdS/CFT analysis cannot give a full account of quantum gravity. If one takes that view, then the results in [39] might be a reflection of the limitations of the AdS/CFT analysis.

## 4. DSE AND BOLTZMANN BRAINS

I now turn to the question of “Boltzmann Brains” in dSE cosmology. The “Boltzmann Brain problem” (first posed in [40] and recently receiving renewed attention[22, 41, 35, 9, 42]) refers to the situation where one’s cosmological framework assigns a higher probability to isolated “observers” fluctuating briefly out of equilibrium versus observers which are correlated with large cosmological (non-equilibrium) states of matter the way we are. Boltzmann Brain observers may have (false) memories of planet Earth, the solar system, galaxies, cosmic microwave background maps etc., but their destiny is to be immediately reabsorbed in the background equilibrium state. Since this is not what we experience, cosmological frameworks which strongly favor Boltzmann Brain observers are typically considered to be failures.

In dSE, the probability assigned to a single Boltzmann Brain observer (BB) is given by using

$$S_{BH} = S_{Br}^S \equiv \left( \frac{m_{Br}}{m_P} \right)^2 \quad (16)$$

in Eqn. 7, where  $S_{Br}^S$  is the entropy of a black hole with the same ADM mass,  $m_{Br}$  as the BB, giving

$$P_{Br} = \exp \left\{ -\sqrt{S_\Lambda S_{Br}} \right\} \quad (17)$$

To compare this with Eqn. 11 for the probability for inflation one must consider the value of  $S_S$ , the entropy of a black hole with ADM mass equal to the seed mass  $m_S$  for the FGG process. While the  $m_S \rightarrow 0$  gives the dominant process, as discussed above and in [22], the thin wall approximation should break down as one takes this limit. Here we assume the scale of the inflaton potential gives a lower bound on  $m_S$  given by

$$m_S = \rho_I R_I^3 = 0.0013 kg \left( \frac{(10^{16} GeV)^4}{\rho_I} \right)^{1/2}. \quad (18)$$



Keeping only the dominant terms gives

$$\frac{P_{Br}}{P_I} = \frac{\exp\{-\sqrt{S_\Lambda S_{Br}}\}}{\exp\{-\sqrt{S_\Lambda S_S}\}}. \quad (19)$$

Inflation is favored when  $S_S < S_{Br}$  which gives

$$\left(\frac{m_{Br}}{0.0013kg}\right) > \left(\frac{(10^{16}GeV)^4}{\rho_I}\right)^{1/2} \quad (20)$$

which is a condition that is very easily met. Thus the dSE framework (using the FG process) does not appear to suffer from the BB problem.

## 5. DSE, ENTROPY AND TIME SYMMETRY

### 5.1. The low entropy of inflation-produced SBB's

From a sufficiently fine-grained point of view, an SBB cosmology that started with reheating at the end of an inflationary era will always have much lower entropy than other SBB cosmologies that don't have that special starting point. This can be illustrated by the fact that the time-reverse of an SBB cosmology is highly unlikely to suck all the energy of the universe into the coherent (and extremely low entropy form) of a homogeneous rolling scalar field.

Although this feature is sometimes cited as a problem for inflation [43, 44, 45], in the context of dSE cosmology things are just as they should be. It is a remarkable property of inflation that it can take a low entropy (inflating) state and reheat it into an state that appears sufficiently high in entropy (from a suitably coarse-grained point of view) to look like an SBB cosmology. But the fact that the inflationary path to the SBB actually has much lower fine grained entropy than the “typical” SBB allows it to leave more entropy in the “heat bath”, making it a much more likely fluctuation out of dSE.

### 5.2. What about the time reversed process?

As discussed already in section 3.1, the late time evolution of our observed universe will eventually re-equilibrate with the eternal de Sitter space. Also, since there is only some small probability of tunneling into inflation in the first place, there will be different parts of the wavefunction representing the part that tunneled into inflation and the part that did not. These different parts will “re-cohere” as they all approach (or remain in) the dSE state.

It seems reasonable to expect that this late-time re-equilibration would look very much the same whether it was an inflation-produced SBB or a “regular” SBB doing the equilibration. Furthermore, with nothing defining a universal time direction for the dSE state, one could equally well expect the “tunneling out” and “equilibrating back” events

to happen in either order (a meta-observer with a universal time arrow would equally often see the time reversed process, but of course a resident of the corresponding SBB phase would just see time in the direction of increasing entropy).

The dSE framework implies that the process of an SBB “decohering out” of the dSE (the time reverse of re-equilibration) should be much more likely if the SBB state in question is destined (due to extremely subtle microscopic features) to eventually back into an inflating state, than if it were a “regular” SBB with no period of inflation at the “beginning”. But it seems extremely hard to imagine how that could be the case, since it seems natural for the re-coherence process to look the same in terms of features that matter to the dynamical properties at that stage (i.e. localized mass concentrations that could change the de Sitter horizon).

My response to this point is that indeed it is hard to see how the probabilities would work out to be so different for the two cases, but we should be used to that sort of thing being hard. A box of gas has a much lower probability of fluctuating in and out of a gold watch state than it does fluctuating in and out of a pile of dirt state. But one would expect the final stages of re-cohering to the equilibrium state (at both ends of the fluctuation) would look very much the same for the cases of the gold watch and the dirt. It is probably very hard to tell at that stage of the evolution what makes one fluctuation much more probable than the other. So I believe if one can tolerate that being a difficult problem for the box of gas, one should be able to tolerate the equivalent problem for the dSE picture being at least as hard.

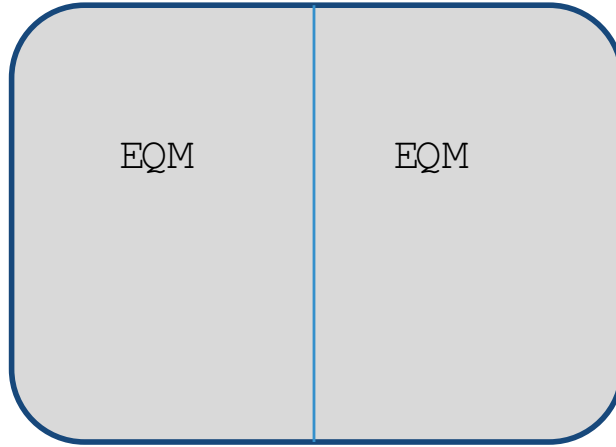
One can also ask whether there is something fundamentally wrong with the FGG process since it looks so time-asymmetrical (tunneling at one end and equilibration at the other). What place is there for such an asymmetric process in system with so much time symmetry? Even the “watch” and “dirt” fluctuations discussed directly above would be expected to be fully time-symmetric.

To further consider this question I propose the following analogy: Consider a box of gas divided in two by a barrier that can only be penetrated by quantum tunneling, as depicted in Fig. 1. The gas on both sides of the barrier is in equilibrium, with the same equilibrium properties on both sides.

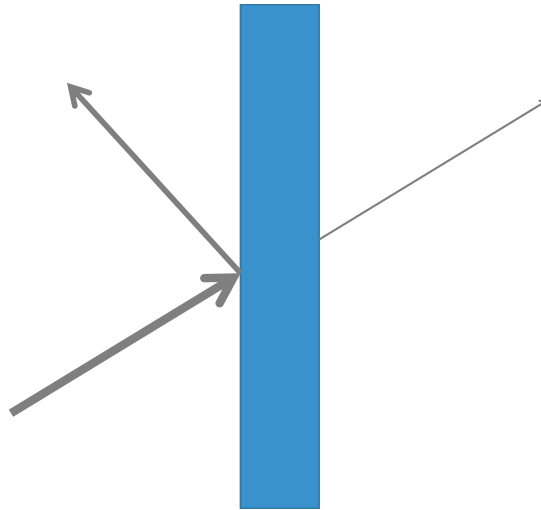
Figure 2 depicts a zoomed-in view of a gas particle tunneling through the barrier. The wavefunction of the incoming particle divides into two pieces, a coherent sum of a piece that tunnels through and a piece that bounces back. The time reverse of this process seems very odd: A particle coherently matched on both sides of the barrier in order to pull onto just one side after striking the barrier. But the box of gas is perfectly time-symmetrical and the tunneling process occurring in both directions is equally likely. I suspect our preference for viewing this tunneling event in a particular time direction is a reflection of our own experience living in a very time-asymmetric world, and I believe a similar assessment applies to concerns about the apparent time-asymmetry of the FGG process in the dSE picture.

## 6. DSE AND THE CLOCK AMBIGUITY

The dSE cosmological framework appears to be built on laws of physics that are truly eternal and stable. One has the picture of a universe evolving though infinitely many



**FIGURE 1.** A box of gas with a barrier that can only be penetrated by quantum tunneling. The gas on both sides is in an equilibrium state with the same properties.



**FIGURE 2.** A zoomed-in cartoon of a tunneling event at the barrier depicted in Fig. 1. We find it convenient to think of the tunneling in this highly time-asymmetric way, even though the system as a whole is time-symmetric.

recurrence times and building up statistics for even the rarest of fluctuations. Apparently all this would require perfectly stable physical laws.

In another line of investigation, Iglesias and I [46, 47, 48] have studied a picture motivated by the “clock ambiguity” in which the laws of physics are emergent and are only stable for a finite period of time. Using arguments developed in [48] (especially Eqn. 8 in that paper) one can find lower bounds on  $N_H$ , the number of states in the Hilbert space describing our physical world. One gets values for the lower bound around  $\exp\{10^{60}\}$  or even  $\exp\{10^{100}\}$ , but typically not higher. In the context of that work it makes sense to regard the lower bound as the typical or possibly even the predicted

value of  $N_H$ . From that point of view,  $N_H$  from the clock ambiguity work would be exponentially smaller than the value  $N_H \geq \exp\{S_\Lambda\} \approx \exp\{10^{120}\}$  required to have stable physical laws even for one recurrence time.

Thus it would seem that these two ways of thinking about the cosmos and physical laws (the clock ambiguity work and the dSE cosmology) are deeply at odds with one another. That in itself is not necessarily a problem, since all these ideas lie in such a speculative domain that this is probably not the right time to expect them to fit together. But still, I find it interesting to reflect a bit further on the possible connections between these ways of thinking.

In the dSE cosmology, the universe spends by far most of the time simply sitting in equilibrium, subject to tiny fluctuations that are completely uninteresting for cosmology. How much stability is required of the laws of physics in order to sustain this equilibrium picture? Perhaps not a lot<sup>3</sup>. Perhaps these two pictures can coexist as long as the laws of physics can be stable over cosmologically interesting timescales, a requirement that we showed in [48] is quite easy to meet.

## 7. CONCLUSIONS

Viewing cosmology as the study of fluctuations around a de Sitter equilibrium state has a number of appealing features. It seems more naturally finite than the picture that emerges from “chaotic inflation” (for which the regulation of infinities is a problem), and does not depend on what one assumes for “initial conditions” (it does not have any). When viewed from the point of view of the FG process (as in [22]), this framework shows that inflation is favored over big bang cosmologies without inflation because inflation presents an entropically “cheaper” way to form cosmological fluctuations. In this paper I have also quantified the claim (first made in [29]) that one of the important roles of inflation can be to evade the “Boltzmann Brain” problem. I’ve also sought to address various questions that have been raised about this picture, in the hopes of moving those discussions forward.

There are many open questions: Which (if either) interpretation of the dSE cosmology (DKS[21] or AS[22]) is correct? (They give totally different answers.) How can one make the treatment more rigorous? (A new formalism for correlation functions in de Sitter space[50] or other explorations[51] may hold some clues.) How can the predictive power be more fully developed? I find these questions interesting in their own right, but especially so given the many attractive features of de Sitter equilibrium cosmology. Progress on these open questions will help us see if these ideas can ultimately make good on their promise to provide deep theoretical foundations for cosmology.

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<sup>3</sup> This is related to, but not quite the same as a point made in [49] about the stability of quantum measurements over a recurrence time

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## REFERENCES

1. A. D. Linde, *Phys. Lett.* **B175**, 395–400 (1986).
2. A. Linde, *JCAP* **0706**, 017 (2007), 0705.1160.
3. A. H. Guth, *J. Phys.* **A40**, 6811–6826 (2007), hep-th/0702178.
4. J. Garriga, and A. Vilenkin, *Phys. Rev.* **D77**, 043526 (2008), 0711.2559.
5. A. Linde, *Lect. Notes Phys.* **738**, 1–54 (2008), 0705.0164.
6. A. Aguirre, S. Gratton, and M. C. Johnson, *Phys. Rev.* **D75**, 123501 (2007), hep-th/0611221.
7. R. Bousso, B. Freivogel, and I.-S. Yang, *Phys. Rev.* **D77**, 103514 (2008), 0712.3324.
8. A. Linde, V. Vanchurin, and S. Winitzki, *JCAP* **0901**, 031 (2009), 0812.0005.
9. A. De Simone, et al. (2008), 0808.3778.
10. A. De Simone, A. H. Guth, M. P. Salem, and A. Vilenkin, *Phys. Rev.* **D78**, 063520 (2008), 0805.2173.
11. R. Bousso, B. Freivogel, and I.-S. Yang, *Phys. Rev.* **D79**, 063513 (2009), 0808.3770.
12. J. Garriga, and A. Vilenkin, *JCAP* **0901**, 021 (2009), 0809.4257.
13. A. J. Albrecht, N. Kaloper, and Y.-S. Song (2002), hep-th/0211221.
14. N. Arkani-Hamed, S. Dubovsky, A. Nicolis, E. Trincherini, and G. Villadoro, *JHEP* **05**, 055 (2007), 0704.1814.
15. S. Dubovsky, L. Senatore, and G. Villadoro, *JHEP* **04**, 118 (2009), 0812.2246.
16. R. Bousso, *Phys. Rev. Lett.* **97**, 191302 (2006), hep-th/0605263.
17. A. Linde, *JCAP* **0701**, 022 (2007), hep-th/0611043.
18. J. B. Hartle, S. W. Hawking, and T. Hertog, *Phys. Rev. Lett.* **100**, 201301 (2008), 0711.4630.
19. J. B. Hartle, S. W. Hawking, and T. Hertog, *Phys. Rev.* **D77**, 123537 (2008), 0803.1663.
20. R. Bousso, and I.-S. Yang (2009), 0904.2386.
21. L. Dyson, M. Kleban, and L. Susskind, *JHEP* **10**, 011 (2002), hep-th/0208013.
22. A. Albrecht, and L. Sorbo, *Phys. Rev.* **D70**, 063528 (2004), hep-th/0405270.
23. S. R. Coleman, and F. De Luccia, *Phys. Rev.* **D21**, 3305 (1980).
24. E. Mottola, *Phys. Rev.* **D31**, 754 (1985).
25. N. C. Tsamis, and R. P. Woodard, *Ann. Phys.* **238**, 1–82 (1995).
26. R. P. Feynman, *The character of physical law [by] Richard Feynman*, M.I.T. Press Cambridge, 1965.
27. S. M. Carroll, and J. Chen (2004), hep-th/0410270.
28. D. N. Page, *Nature* **304**, 39–41 (1983).
29. A. Albrecht (2004), in *Science and Ultimate Reality: From Quantum to Cosmos*, honoring John Wheeler’s 90th birthday. J. D. Barrow, P.C.W. Davies, & C.L. Harper eds. Cambridge University Press (2004), astro-ph/0210527.
30. T. Banks, and W. Fischler, *Phys. Scripta* **T117**, 56–63 (2005), hep-th/0310288.
31. R. Bousso, R. Harnik, G. D. Kribs, and G. Perez, *Phys. Rev.* **D76**, 043513 (2007), hep-th/0702115.
32. G. W. Gibbons, and S. W. Hawking, *Phys. Rev.* **D15**, 2738–2751 (1977).
33. E. Farhi, and A. H. Guth, *Phys. Lett.* **B183**, 149 (1987).
34. E. Farhi, A. H. Guth, and J. Guven, *Nucl. Phys.* **B339**, 417–490 (1990).
35. T. Banks (2007), hep-th/0701146.
36. A. Aguirre, and M. C. Johnson, *Phys. Rev.* **D73**, 123529 (2006), gr-qc/0512034.
37. W. Fischler, D. Morgan, and J. Polchinski, *Phys. Rev.* **D41**, 2638 (1990).
38. W. Fischler, D. Morgan, and J. Polchinski, *Phys. Rev.* **D42**, 4042–4055 (1990).
39. B. Freivogel, et al., *JHEP* **03**, 007 (2006), hep-th/0510046.

40. L. Boltzmann, *Nature* **51**, 413–415 (1895).
41. D. N. Page (2006), [hep-th/0611158](#).
42. L. Mersini-Houghton, and F. C. Adams, *Class. Quant. Grav.* **25**, 165002 (2008), [0810.4914](#).
43. R. Penrose, *Annals N. Y. Acad. Sci.* **571**, 249–264 (1989).
44. W. G. Unruh (1997), in *Critical Dialogs in Cosmology*, N. Turok ed., p249, World Scientific (1997).
45. S. Hollands, and R. M. Wald (2002), [hep-th/0210001](#).
46. A. Albrecht (1994), [gr-qc/9408023](#).
47. A. J. Albrecht, and A. Iglesias, *Phys. Rev.* **D77**, 063506 (2008), [0708.2743](#).
48. A. J. Albrecht, and A. Iglesias (2008), [0805.4452](#).
49. T. Banks, W. Fischler, and S. Paban, *JHEP* **12**, 062 (2002), [hep-th/0210160](#).
50. B. Freivogel, and M. Kleban (2009), [0903.2048](#).
51. T. Hertog, and G. T. Horowitz, *JHEP* **04**, 005 (2005), [hep-th/0503071](#).